# THE ELLIPTIC ENERGY LOSS FOR ROTATED OBJECT DETECTION IN AERIAL IMAGES

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## ABSTRACT

Rotated object detection is a promising yet challenging task in computer vision. Existing algorithms mainly train the rotated detector by the  $L_n$ -norm loss, which is inconsistent with the evaluation metric of Intersection over Union (IoU). However, the concise and efficient solution using the loss based on IoU between oriented boxes is hindered by its nondifferentiability. In this paper, we propose Elliptic Energy Loss, a differentiable loss based on curve energy, to fit with the evaluation metric IoU. Specifically, given a pair of predicted and ground truth boxes, we first convert them to curve representations using the elliptic transformation. Then, the curve energy is calculated to measure the similarity between the predicted and ground truth curves. Finally, the curve energy is used as regression loss to optimize rotated detectors. We conduct experiments with different detectors on DOTA and HRSC2016 datasets, which demonstrate that the performance is significantly improved by our proposed loss. The code is available at https://github.com/zhangc-uestc/EEL.

Index Terms- Object detection, rotated object detection, regression loss

## 1. INTRODUCTION

Rotated object detection is a fundamental task in computer vision, with various applications in environmental change monitoring [1] and intelligent transportation [2]. Compared with horizontal object detection [3-5], rotated object detection regresses oriented bounding boxes (OBBs) that locate objects more tightly, which exhibits apparent advantages when detecting objects with arbitrary orientation and dense arrangements [6-8]. To train rotated detectors, existing algorithms mainly use the  $L_n$ -norm loss as the regression loss. However, the  $L_n$ -norm loss fails to accurately reflect the change in the evaluation metric Intersection over Union (IoU) [9, 10], which limits the performance of rotated detectors. As is demonstrated in horizontal detection, using



Fig. 1. Comparison of our Elliptic Energy Loss with ideal IoU loss and existing losses such as L1, Smooth L1 [3], GWD [15], KLD [16], KFIoU [10]. Given predicted and ground truth OBBs, we compute the loss values of (a) angle difference and (b) center offset.

IoU loss or its variants [11-13] can effectively solve this inconsistency problem. Unfortunately, these solutions cannot be applied to rotated detection since the calculation of IoU between OBBs (called SkewIoU [14]) is non-differentiable.

To narrow the gap between the regression loss and the evaluation metric, IoU-Smooth L1 loss [17] takes the negative logarithm of SkewIoU as the magnitude of the gradient directly, PIoU [9] estimates the intersection of two OBBs in a pixel-wise manner. These methods improved the accuracy of localization, but the inconsistency problem is only par-

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tially solved. Recently, Gaussian-based losses (GWD [15], KLD [16], KFIoU [10]) are proposed to mimic the mechanism of SkewIoU using Gaussian distribution, which achieves superior performance. Despite their success, as illustrated in Fig. 1, Gaussian-based losses do not completely fit the IoU loss. In the center offset case, GWD and KLD tend to be constant as the distance increases. It is also the flaw of IoU loss, which is suboptimal for initial learning of detectors.

In this paper, we propose a new regression loss called Elliptic Energy loss, which approximates the non-differentiable SkewIoU using a differentiable similarity metric called elliptic energy. Specifically, we first convert the predicted OBB and the ground truth OBB to their inscribed ellipse representations. Then we use the energy function inspired by the active contour model [18] to measure the similarity between the predicted curve and the ground truth curve. Since the curve transformation and the energy calculation are differentiable, they can be applied as the regression loss. As shown in Fig. 1, our loss is the closest approximation to SkewIoU compared with other losses. We conduct various experiments on two popular datasets, DOTA and HRSC2016, to validate the effectiveness of our method, achieving obvious gains over the baseline and state-of-the-art performance.

## 2. PROPOSED METHOD

Rotated detection aims at locating objects in images using OBBs that are typically represented by 5 parameters ( $x_c$ ,  $y_c$ , w, h,  $\theta$ ), which stand for the center of the object, width (long side), height (short side), and the angle from the positive direction of the x-axis to the direction parallel to w in the Long Edge definition [14]. To train the rotated detector, regression loss is required to measure the difference between the predicted and ground truth OBBs. In this paper, we propose Elliptic Energy loss, in which we first convert OBBs into elliptic curve representations (Sec 2.1), and then calculate the energy of curves (Sec 2.2), which can finally be used as the regression loss (Sec 2.3) that fits the SkewIoU well.

#### 2.1. Curve representation of the object

Given two OBBs, we transform them into their curve representations, which should be continuous and differentiable. Thus, we propose to describe the object using the inscribed elliptic curve of OBB, which can be formulated as follows:

$$f_{rt}(x,y) = \frac{[(x - x_c)\cos\theta + (y - y_c)\sin\theta]^2}{(w/2)^2} + \frac{[(y - y_c)\cos\theta - (x - x_c)\sin\theta]^2}{(h/2)^2}$$
(1)  
= 1,

where  $x_c, y_c, w, h, \theta$  are parameters of OBB, and (x, y) is the point on the ellipse if  $f_{rt}(x, y) = 1$ . Similarly, the relative



**Fig. 2.** Illustration of the curve energy. Mask is constructed in terms of the ground truth curve T. The internal energy  $E_{in}(C)$  in Eq. (2) comes from the yellow region, where  $c_1$ is the average of the yellow region. The external energy  $E_{out}(C)$  comes from the blue region, where  $c_2$  is the average of the blue region.



**Fig. 3.** Diagram of normalization. For given two boxes, OBB<sub>1</sub> and OBB<sub>2</sub>, we first find their external horizontal box HBB (the smallest enclosing box covering two boxes) and its relevant parameters including the upper left corner  $(x_{ul}, y_{ul})$ , the lower right corner  $(x_{lr}, y_{lr})$ , the width w and the height h. Then we calculate the scale factor *scale*, and the vertex coordinates  $(x_{new}, y_{new})$  of the new boxes using *scale* and  $(x_{ul}, y_{ul})$ , so that the new boxes are limited to  $tsize \times tsize$ .

position from any point (x, y) to the ellipse can be determined by  $f_{rt}(x, y)$ . In this way, object representation is transformed from the bounding box to the elliptic curve, which facilitates the similarity measure between curves later.

## 2.2. Similarity measure between curves

After elliptic transformation of OBBs, we then measure the similarity between the predicted curve C and the ground truth curve T. Inspired by the active contour model [18], we weigh the similarity with customized energy, which contains two fitting items as Eq. (2).

$$E(C) = E_{in}(C) + E_{out}(C)$$
  
= 
$$\int_{inside(C)} |M_T(x, y) - c_1|^2 dx dy$$
  
+ 
$$\int_{outside(C)} |M_T(x, y) - c_2|^2 dx dy,$$
 (2)

in which

$$M_T(x,y) = \begin{cases} 1, & \text{if } f_{rt}^T(x,y) \le 1\\ 0, & \text{otherwise.} \end{cases}$$
(3)

where  $E_{in}(C)$  and  $E_{out}(C)$  represent the internal and external energies of C on  $M_T$ , respectively.  $M_T$  is a special image we construct with T that corresponds to the input image in the active contour model, where pixel values inside and including T are 1 and pixel values outside T are 0, as shown in Fig. 2. The constants  $c_1, c_2$  represent the averages of  $M_T$ inside and outside C, respectively. Eq. (2) shows that if Cis identical to T or completely inside  $T, E_{in}(C) = 0$ , otherwise  $E_{in}(C) > 0$ , which indicates that minimizing the internal energy tightens the curve C inwards. And if C is identical to T or completely outside  $T, E_{out}(C) = 0$ , otherwise  $E_{out}(C) > 0$ , which means that minimizing the external energy stops the curve C when C shrinks to T. Finally, fitting energy is minimized when C = T.

However, we observe that Eq. (3) is not a continuous and differentiable function. It will break the back propagation of the gradient if we use Eq. (3) directly. Inspired by PIoU [9], we update Eq. (3) based on a kernel function:

$$M(x,y) \approx K(f_{rt}(x,y),1) \tag{4}$$

in which

$$K(d,s) = 1 - \frac{1}{1 + e^{-k(d-s)}}$$
(5)

where K(d, s) is the kernel function, k(>0) is an adjustable factor that controls the sensitivity. K(d, s) tends to 0 when d > s, and tends to 1 when d < s. Thus,  $K(f_{rt}(x, y), 1)$ can be used as a continuous and differentiable alternative to Eq. (3).

To simplify the calculation, we also construct the mask  $M_C$  corresponding to the predicted curve C, so that Eq. (2) can be computed as follows:

$$E(C) = \sum_{x,y} \left[ M_C(x,y) \times (M_T(x,y) - c_1) \right]^2 + \sum_{x,y} \left[ (1 - M_C(x,y)) \times (M_T(x,y) - c_2) \right]^2$$
(6)

in which

$$c_{1} = \frac{\sum_{x,y} M_{C}(x,y) \times M_{T}(x,y)}{\sum_{x,y} M_{C}(x,y)}$$
(7)

$$c_{2} = \frac{\sum_{x,y} (1 - M_{C}(x,y)) \times M_{T}(x,y)}{W \times H - \sum_{x,y} M_{C}(x,y)}$$
(8)

where W(H) represents the width (height) of the  $M_C$ , which is also the width (height) of the input image and  $M_T$ .  $x \in [0, W), y \in [0, H)$ . By this way, the similarity between the predicted and the ground truth curve can be measured using defined energy, i.e., Eq. (6)-(8).

#### 2.3. Regression loss

Note that the calculation above is done pixel by pixel. To lower the consumption of computational resources, we suggest normalizing oriented boxes first. The specific steps are depicted in Fig. 3, where *tsize* is the size after normalizing and is generally set to 50 or 100. It further ensures scale invariance of the loss with rescaling operations. Following normalization, all pixel-wise calculations are restricted to the range of  $tsize \times tsize$ , thus only  $tsize \times tsize$  is needed to construct the mask, that is,  $x \in [0, tsize), y \in [0, tsize)$ . But the normalization method shown in Fig. 3 introduces a new problem. Assuming that the predicted box is far away from the ground truth box, their external horizontal box will be large, resulting in the predicted and ground truth boxes becoming small after rescaling. The slight difference between  $M_T$  and  $M_C$  will result in a small loss. This means that the current method is not suitable for measuring predicted and ground truth boxes that are far apart, as the ideal loss should increase with their distance. To fix this problem, we add the distance between the centers of the predicted and ground truth boxes to the loss as center loss, so that the center loss takes the lead when the two boxes are far away, and the energy takes the lead otherwise.

We select Smooth L1 loss as the center loss, and condense energy using a linear function as the energy loss, which are indicated by Eq. (9) and Eq. (10).

$$L_{ct}(t,t^*) = \frac{1}{2} \sum_{j \in \{x,y\}} \text{Smooth } L1(t_j - t_j^*)$$
(9)

$$L_e(b,gt) = \lambda E(C,T) \tag{10}$$

where  $t(t^*)$  denotes the offset from the center of the predicted box (the ground truth box) to the center of the anchor box.  $t_x = \frac{x_c - x'_c}{w'}, t_y = \frac{y_c - y'_c}{h'}, t_x^* = \frac{x_c^* - x'_c}{w'}, t_y^* = \frac{y_c^* - y'_c}{h'}$ .  $x_c, y_c, w, h$  denote the center coordinates, width, and height of the box, respectively. Variables  $x_c, x'_c, x_c^*$  are for the predicted box, anchor box, and ground-truth box (likewise for  $y_c, w, h$ ). b and gt represent the predicted box and ground truth box, respectively.  $\lambda$  is hyper-parameter determined by follow-up experiments.

The entire regression loss is as follows:

$$L = \frac{1}{N_{pos}} \sum_{i=1}^{N_{pos}} L_{ct} \left( t_i, t_i^* \right) + L_e \left( b_i, g t_i \right)$$
(11)

where  $N_{pos}$  indicates the number of positive samples.

#### 3. EXPERIMENTS

#### 3.1. Datasets and Implementation Details

We conduct experiments on DOTA [19] and HRSC2016 [2], which are the most common datasets for oriented object detection. All experiments are implemented by MMRotate [20]

**Table 1.** Ablation study of hyperparameters with the RetinaNet-HBB detector on DOTA. The **bold** and <u>underlined</u> fonts indicate the top two performances, respectively. The metric is  $mAP_{50}$ .

Parameter	$\lambda = 1/500$	$\lambda = 1/1000$	$\lambda = 1/2000$
k = 10	68.85	70.31	<u>70.05</u>
k = 100	68.19	69.50	69.76
k = 1000	66.89	68.57	68.63

Table 2. Ablation study of hyperparameters with the RetinaNet-HBB detector on HRSC2016. The metrics used are  $AP_{50}$  /  $AP_{75}$  /  $mAP_{50:95}$ .

Parameter	$\lambda = 1/500$	$\lambda = 1/1000$	$\lambda = 1/2000$
k = 10	86.8 / 60.3 / 54.49	<b>87.4</b> / 68.9 / 57.75	86.7 / <b>70.9</b> / <b>59.60</b>
k = 100	85.6 / 60.3 / 54.02	<u>87.0</u> / <u>69.1</u> / 56.22	86.6 / <u>69.1</u> / <u>58.69</u>
k = 1000	84.9 / 53.5 / 48.58	86.3 / 59.7 / 53.73	85.7 / 68.9 / 56.99

**Table 3**. Comparisons with peer losses under different detectors on DOTA. The definition methods of oriented boxes on both detectors are OpenCV Definition ( $\theta \in (0, \frac{\pi}{2}]$ ). The values in brackets show the gain or reduction of each loss relative to the Smooth L1 loss.

Model	Reg.Loss	mAP <sub>50</sub>
	Smooth L1	65.08
	GWD [15]	68.70(+3.62)
RetinaNet-HBB [5]	KLD [16]	69.39(+4.31)
	KFIoU [10]	69.48(+4.40)
	EEL (Ours)	70.31(+5.23)
	Smooth L1	69.01
	GWD [15]	72.42(+3.41)
$R^{3}Det$ [8]	KLD [16]	72.73(+3.72)
	KFIoU [10]	72.25(+3.24)
	EEL (Ours)	73.15(+4.14)

 Table 4. Comparisons with peer losses under different detectors on HRSC2016.

Model	Reg.Loss	$AP_{50}$	$AP_{75}$	$mAP_{50:95}$
	Smooth L1	81.3	54.5	48.18
	GWD [15]	84.9(+3.6)	67.1(+12.6)	55.98(+7.8)
RetinaNet-HBB [5]	KLD [16]	86.8(+5.5)	69.6(+15.1)	58.48(+10.3)
	KFIoU [10]	84.7(+3.4)	60.4(+5.9)	54.61(+6.43)
	EEL (Ours)	86.7(+5.4)	70.9(+16.4)	59.60(+11.42
	Smooth L1	86.2	57.6	52.33
	GWD [15]	87.0(+0.8)	<b>69.7(+12.1)</b>	58.55(+6.22)
$R^{3}Det$ [8]	KLD [16]	87.5(+1.3)	68.2(+10.6)	55.92(+3.59)
	KFIoU [10]	86.5(+0.3)	68.8(+11.2)	57.51(+5.18)
	EEL (Ours)	88.5(+2.3)	68.2(+10.6)	55.77(+3.44)

using one TITAN Xp GPU. We set batch size to 2 and train models on DOTA and HRSC2016 for 12 and 72 epochs, respectively. For fair comparison, we use the same configuration to train the models, except with different losses.

## 3.2. Ablation Studies

In Table 1 and Table 2, we conduct ablation experiments on the hyperparameter settings of k and  $\lambda$  in our elliptic energy



**Fig. 4**. Qualitative comparison between the Smooth L1 loss and the proposed Elliptic Energy loss with same RetinaNet-HBB detector on DOTA and HRSC2016.

loss on DOTA and HRSC2016 datasets. As can be seen, performance drops rapidly when k increases. The reason may be that a larger k will cause a larger gradient of the loss, making the loss more sensitive to prediction errors during training. Also, choosing a smaller  $\lambda$  can improve performance when given a larger k. As suggested by Table 1 and Table 2, we set  $k = 10, \lambda = 1/1000$  on DOTA, and  $k = 10, \lambda = 1/2000$  on HRSC2016 for subsequent experiments.

### 3.3. Comparisons with previous methods

In Table 3 and Table4, we compare our method with previous regression losses, such as Smooth L1 [3], GWD [15], KLD [16] and KFIoU [10]. We train RetinaNet-HBB [5] and R<sup>3</sup>Det [8] detectors on DOTA and HRSC2016 datasets using different regression losses. Smooth L1 loss is used as the baseline. As the results show, on DOTA dataset, our method achieves mAP<sub>50</sub> = 70.31% and mAP<sub>50</sub> = 73.15% using RetinaNet-HBB and R<sup>3</sup>Det, which is the best compared with previous methods. On HRSC2016 dataset, our method can also produce superior results of AP<sub>75</sub> = 70.9% and mAP<sub>50:95</sub> = 59.60% using RetinaNet-HBB and AP<sub>50</sub> = 88.5% using R<sup>3</sup>Det. The qualitative comparison between the Smooth L1 loss and our proposed loss is shown in Fig. 4. As a result, the model trained by our loss can locate objects more precisely than the model trained by the Smooth L1 loss.

## 4. CONCLUTIONS

In order to alleviate the inconsistency between the regression loss and the evaluation metric, this paper proposes the Elliptic Energy loss. The loss first converts the paired predicted box and the ground truth box into their corresponding curve representations, and then utilizes the curve energy to assess how similar the predicted curve and the ground truth curve are. Additionally, by transforming it appropriately and increasing the center loss, it can be used for training. Numerous experiments on DOTA and HRSC2016 demonstrate the effectiveness of our loss.

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